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Analytical Surface Roughness Parameters of a Theoretical Profile Consisting of Elliptical Arcs

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ABSTRACT

The closed-form solutions of surface roughness parameters for a theoretical profile consisting of elliptical arcs are presented. Parabolic and simplified approximation methods are commonly used to estimate the surface roughness parameters for such machined surface profiles. The closed-form solution presented in this study reveals the range of errors of approximation methods for any elliptical arc size. Using both implicit and parametric methods, the closed-form solutions of three surface roughness parameters, R_t, R_a, and R_q, were derived. Their dimensionless expressions were also studied and a single chart was developed to present the surface roughness parameters. This research provides a guideline on the use of approximate methods. The error is smaller than 1.6% when the ratio of the feed and major semi-axis of the elliptical arc is smaller than 0.5. The closed-form expressions developed in this study can be used for the surface roughness modeling in CAD/CAM simulations.

Key Words: Surface roughness; Surface with circular and elliptical arcs.

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INTRODUCTION

Machined surfaces with a cross-section profile consisting of elliptical or circular arcs are commonly generated in turning using a stationary or self-powered rotary tool with a radius tool tip (Cheung and Lee, 2001; El-wardany et al., 1992; Hasegawa et al., 1976; Lambert, 1961–1962; Nassirpour and Wu, 1977; Olsen, 1968; Sata, 1964; Sata et al., 1985; Shaw and Crowell, 1965; Shiraishi and Sato, 1990; Vajpayee, 1981; Wallbank, 1979), ball-end and flat-end milling (Lee, 1998; Lee and Chang, 1996), cylindrical wire EDM (Qu et al., 2002), and other manufacturing processes. The finish and functional performance of a machined surface are characterized and quantified by the surface roughness parameters (De-Chiffre et al., 2000; Malburg et al., 1993; Nowicki, 1981; Thomas, 1981; Vajpayee, 1973; Whitehouse, 1994). Approximation methods using a parabolic curve to match the elliptical or circular arc are commonly used to estimate the surface roughness parameters. To the best of our knowledge, closed-form expressions of the arithmetic average roughness R_a and root-mean-square roughness R_q, two commonly used surface roughness parameters, for the machined surface profile consisting of elliptical arcs have not been reported.

In this study, the close-form solutions for three roughness parameters, the peak-to-valley, arithmetic average, and root-mean-square roughness, R_t, R_a, and R_q, are derived for an ideal machined surface profile consisting of elliptical arcs. The R_t for a surface profile consisting of circular arcs has been well studied since the 1960s (Cheung and Lee, 2001; Lambert, 1961–1962; Olsen, 1968; Sata, 1964; Shaw and Crowell, 1965; Shiraishi and Sato, 1990; Vajpayee, 1981; Whitehouse, 1994). The approximate estimations of R_a for a theoretical profile formed by circular arcs have also been reported (Lee, 1998; Lee and Chang, 1996). The circular arc can be considered as a special case of the elliptical arc with the same length of two semi-axes.

The real surface generated in machining processes and the profile measured by a surface finish measurement machine will deviate from this theoretical surface profile. The tool wear, vibration of the tool during machining, elastic deformation and recovery of the tool and workpiece, build-up edge at tool tip, resolution of the measurement machine, etc. all create deviations from the ideal, theoretical surface profile. Formulas derived in this study are based purely on geometric considerations. This theoretical model can be used as prediction of the surface finish and comparison with the measured surface roughness values to investigate the influence of the work-material, tool behavior, cutting parameters, and other effects on the machined surface.

There are several benefits and applications of such closed-form surface roughness parameters. First, the closed-form solution can be used to evaluate different approximate solutions. The error analysis reveals the limitations and suitable ranges of approximate solutions. Second, it can be used for surface roughness prediction and evaluation in CAD/CAM modeling of machining processes. Compared to the approximate solutions, the closed-form solution offers accurate results for the whole range of process conditions. It is especially beneficial for precision machining processes, such as single-point diamond turning and cylindrical wire EDM. Third, using the dimensionless form, a single chart can be used to show the surface roughness results of theoretical surface profiles with any size of elliptical arcs.

In this paper, the closed-form expressions of the three roughness parameters R_t, R_a, and R_q for the theoretical surface profile consisting of elliptical arcs are first derived.

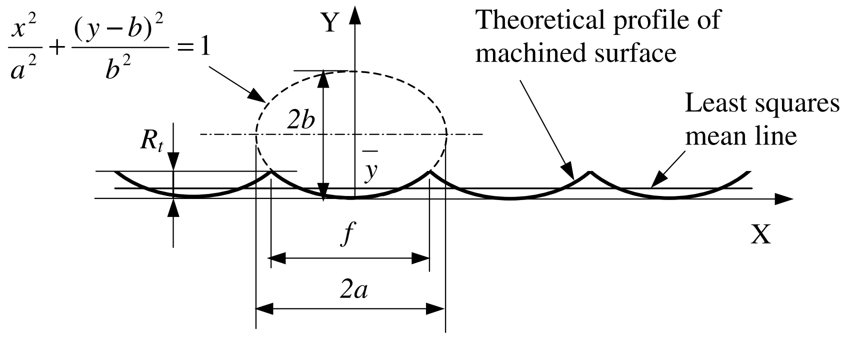

Figure 1. A theoretical surface profile consisting of elliptical arcs.

The surface roughness parameters in dimensionless form are presented in Sec. 3. In Sec. 4, two approximate solutions used to estimate the R_t, R_a, and R_q, are introduced and compared with the closed-form solutions.

CLOSED-FROM EXPRESSIONS OF ROUGHNESS PARAMETERS FOR A THEORETICAL SURFACE PROFILE CONSISTING OF ELLIPTICAL ARCS

Figure 1 illustrates a theoretical surface profile consisting of elliptical arcs. An X-Y coordinate system is defined as shown in Figure 1. The elliptical arc on the surface profile can be expressed as

$$\frac{x^2}{a^2}+\frac{(y-b)^2}{b^2} = 1 \tag{1}$$

where a and b are the major and minor semi-axis of the elliptical arc, respectively, and

$$-\frac{f}{2} \leq x \leq \frac{f}{2} \tag{2}$$

The feed, f, as shown in Figure 1, is the distance between two adjacent peaks of the surface profile. The closed-form expressions of three surface roughness parameters, R_t, R_a, and R_q, for this surface profile with elliptical arcs are derived in the following three sections. The calculation of three parameters, a, b, and f, for a cross-section profile on the surface machined by flat-end milling is presented in Appendix A.

Peak-to-Valley Roughness R_t

The peak-to-valley surface roughness, R_t, of the theoretical profile, as defined in Figure 1, can be expressed as

$$R_t = b-\frac{b}{a}\sqrt{a^2-\frac{f^2}{4}} \tag{3}$$

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Arithmetic Average Roughness R_a

According to ASME B46.1 (1995) and ISO 4287 (1997), the reference mean line in surface roughness is either the least squares mean line or filtered mean line. The least squares mean line is selected. The least squares mean line of this theoretical profile is a straight line parallel to the X-axis, $y = \bar{y}$. Based on the definition of least squares mean line,

$$\frac{\partial \int_{-f/2}^{f/2} (y - \bar{y})^2 dx}{\partial \bar{y}} = 0 \tag{4}$$

Thus,

$$\bar{y} = \frac{1}{f} \int_{-f/2}^{f/2} y dx \tag{5}$$

In order to simplify the expressions of $\bar{y}$, R_a, and R_q, a parameter $S(x)$ is first defined.

$$S(x) = \int y dx = \int \left(b - \frac{b}{a}\sqrt{a^2 - x^2} \right) dx$$
$$= bx - \frac{b}{2a} x\sqrt{a^2 - x^2} - \frac{ab}{2} \arcsin\left(\frac{x}{a}\right) \tag{6}$$

Using $S(x)$, $\bar{y}$ can be expressed as

$$\bar{y} = \frac{2}{f} \int_0^{f/2} y dx = \frac{2}{f}\left[S\left(\frac{f}{2}\right) - S(0) \right] = \frac{2}{f} S\left(\frac{f}{2}\right) \tag{7}$$

Define another parameter x_c as the x coordinate where $y(x_c) = \bar{y}$ on the theoretical profile.

$$x_c = \frac{a}{b}\sqrt{2b\bar{y} - \bar{y}^2} \tag{8}$$

The arithmetic average surface finish R_a is defined as

$$R_a = \frac{1}{f} \int_{-f/2}^{f/2} |y - \bar{y}| dx \tag{9}$$

For the surface profile consisting of elliptical arcs shown in Figure 1, the closed-form expression of R_a can be derived and simplified with the pre-defined parameters, x_c, $\bar{y}$, and $S(x)$.

$$R_a = \frac{2}{f}\left[\int_0^{x_c} (\bar{y} - y) dx + \int_{x_c}^{f/2} (y - \bar{y}) dx \right] = \frac{2}{f}[x_c \cdot \bar{y} - S(x_c)] \tag{10}$$

where x_c, $\bar{y}$, and $S(x_c)$ are functions of f.

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Root-Mean-Square Roughness R_q

The root-mean-square roughness, R_q, is defined as

$$R_q = \sqrt{\frac{1}{f}\int_{-f/2}^{f/2}(y-\bar{y})^2 dx} \tag{11}$$

The closed-form expression of R_q for the theoretical surface profile shown in Figure 1 can be represented by

$$R_q = \sqrt{\frac{2}{f}\int_{0}^{f/2}(y^2-2y\bar{y}+\bar{y}^2)dx} = \sqrt{-\bar{y}^2+2b\bar{y}-\frac{f^2b^2}{12a^2}} \tag{12}$$

Instead of using the above implicit form, another method using the parametric form to derive the closed-form solution of the surface roughness parameters are summarized in the Appendix B. These two methods give different expressions but same results for the three surface roughness parameters.

ROUGHNESS PARAMETERS IN DIMENSIONLESS FORM

In this section, the dimensionless form expressions of three surface roughness parameters are presented. The three dimensionless parameters, R_t/b, R_a/b, and R_q/b, are expressed as functions of another dimensionless parameter, f/a, which ranges from 0 to 2. Using the dimensionless form, a single chart can be used to show the surface roughness results of theoretical surface profiles with any size of elliptical arcs. Different tools, ranging from sharp single-point diamond turning tools with 0.1 mm tool radius, to conventional turning tools with 1 mm tool nose radius and to ball end-milling tools with 5 mm tool radius, can generate elliptical or circular arcs of various sizes.

Peak-to-Valley Roughness R_t

Eq. 3 is rearranged to present the dimensionless peak-to-valley roughness, R_t/b, as a function of f/a.

$$\frac{R_t}{b} = 1-\sqrt{1-\frac{1}{4}\left(\frac{f}{a}\right)^2} \tag{13}$$

Similar dimensionless expression for a theoretical surface profile consisting of circular arcs has been derived by Shaw and Crowell (Shaw and Crowell, 1965).

Arithmetic Average Roughness R_a

The $S(x_c)$, $\bar{y}$, and x_c in Eqs. 6, 7, and 8 are first rewritten as functions of the dimensionless parameter, f/a.

$$\bar{y} = \frac{2}{f} S\left(\frac{f}{2}\right) = b \cdot f_1\left(\frac{f}{a}\right) \tag{14}$$

$$x_c = \frac{a}{b}\sqrt{2b\bar{y} - \bar{y}^2} = a \cdot f_2\left(\frac{f}{a}\right) \tag{15}$$

$$S(x_c) = bx_c - \frac{b}{2a} x_c \sqrt{a^2 - x_c^2} - \frac{ab}{2}\arcsin\left(\frac{x_c}{a}\right) = ab \cdot f_3\left(\frac{f}{a}\right) \tag{16}$$

where

$$f_1\left(\frac{f}{a}\right) = 1 - \frac{1}{2}\sqrt{1 - \frac{1}{4}\left(\frac{f}{a}\right)^2} - \frac{a}{f}\arcsin\left(\frac{f}{2a}\right) \tag{17}$$

$$f_2\left(\frac{f}{a}\right) = \sqrt{2f_1\left(\frac{f}{a}\right) - f_1^2\left(\frac{f}{a}\right)} \tag{18}$$

$$f_3\left(\frac{f}{a}\right) = f_2\left(\frac{f}{a}\right) - \frac{1}{2} f_2\left(\frac{f}{a}\right)\sqrt{1 - f_2^2\left(\frac{f}{a}\right)} - \frac{1}{2}\arcsin\left(f_2\left(\frac{f}{a}\right)\right) \tag{19}$$

Substituting Eqs. 14–16 into Eq. 10, R_a/b can be expressed in the dimensionless form.

$$\frac{R_a}{b} = \frac{1}{b}\left[\frac{2}{f}(x_c\bar{y} - S(x_c))\right] = f_a\left(\frac{f}{a}\right) \tag{20}$$

where

$$f_a\left(\frac{f}{a}\right) = 2\frac{a}{f} \cdot \left[f_1\left(\frac{f}{a}\right) f_2\left(\frac{f}{a}\right) - f_3\left(\frac{f}{a}\right)\right] \tag{21}$$

Root-Mean-Square Roughness R_q

Substituting $\bar{y}$ in Eq. 14 to Eq. 12, R_q/b can be expressed as

$$\frac{R_q}{b} = \frac{1}{b}\sqrt{-\bar{y}^2 + 2b\bar{y} - \frac{f^2 b^2}{12a^2}} = f_q\left(\frac{f}{a}\right) \tag{22}$$

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where

$$f_q\left(\frac{f}{a}\right) = \sqrt{-f_1^2\left(\frac{f}{a}\right) + 2f_1\left(\frac{f}{a}\right) - \frac{1}{12}\left(\frac{f}{a}\right)^2} \tag{23}$$

A single chart, Figure 2, can present the three dimensionless surface roughness parameters, R_t/b, R_a/b, and R_q/b, vs. f/a for a theoretical surface profile consisting of any size elliptical arcs. Knowing the value of f/a, Figure 2, as a quick reference, can be used to get approximate values of the corresponding R_t/b, R_a/b, and R_q/b. The accurate R_t/b, R_a/b, and R_q/b can be calculated by Eqs. 13–23. These three dimensionless values are then multiplied by b to obtain values for surface roughness parameters, R_t, R_a, and R_q. Two examples are described below.

Example 1. Flat-end milling with 5 mm radius tool. Assume the major semi-axis a, minor semi-axis b, and feed f, equal 5, 2, and 2 mm, respectively, then $f/a = 0.4$. Using Eqs. 13–23, the corresponding $R_t/b = 0.0202$, $R_a/b = 0.0052$, and $R_q/b = 0.0060$. By multiplying $b = 2$ mm to these three dimensionless parameters, the R_t, R_a, and R_q, of this theoretical surface profile are 40.4, 10.4, and 12.0 μm, respectively.

Example 2. Single-point diamond turning with 0.1 mm radius tool. In this case, the theoretical surface profile is formed by circular arcs with $a = b = r = 0.1$ mm. Assume the feed $f = 0.008$ mm, the $f/a = 0.08$. The corresponding $R_t/b = 0.00080$, $R_a/b = 0.00021$, and $R_q/b = 0.00024$. For $b = 0.1$ mm, R_t, R_a, and R_q are 0.80, 0.21, and 0.24 μm, respectively.

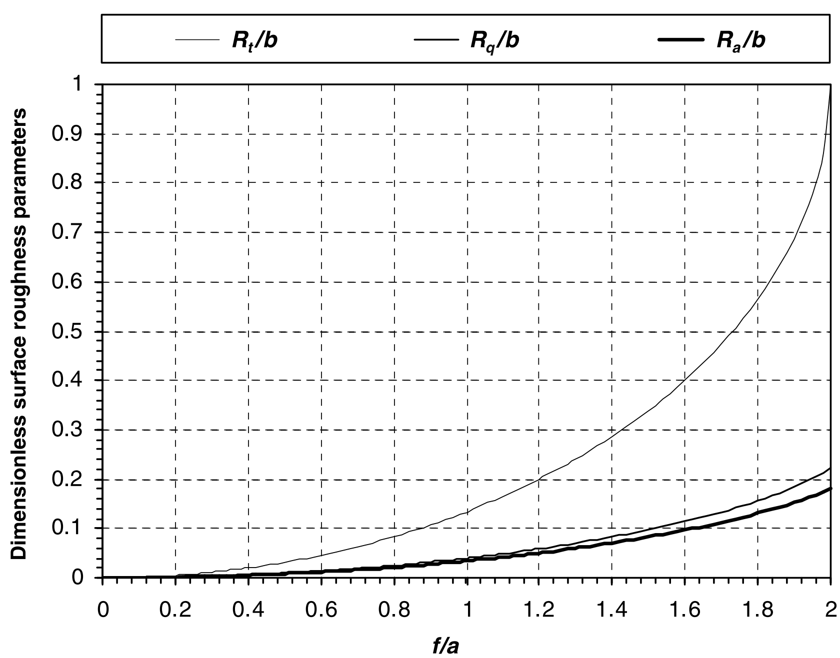

Figure 2. Dimensionless surface roughness parameters R_t/b, R_q/b, and R_a/b vs. f/a.

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APPROXIMATE SOLUTIONS

Approximate solutions of the three roughness parameters for the theoretical surface profile consisting of elliptical arcs are presented. Two approximate solutions, one based on the parabolic approximation and another further simplified approximation, are derived. Results and comparisons to closed-form solutions are given in the following sections.

Approximations of the Peak-to-Valley Roughness R_t

Assuming R_t is small and the higher order term R_t^2 can be neglected, the peak-to-valley roughness R_t of the theoretical profile consisting of elliptical arcs can be simplified to

$$R_t \cong \frac{f^2 b}{8a^2} \tag{24}$$

Substituting a and b by r in Eq. 24, the peak-to-valley roughness R_t of a theoretical surface profile consisting of circular arcs is $R_t \cong f^2/8r$, which has been reported in references (Cheung and Lee, 2001; Lambert, 1961–1962; Olsen, 1968; Sata, 1964; Shaw and Crowell, 1965; Shiraishi and Sato, 1990; Vajpayee, 1981; Whitehouse, 1994). Vajpayee (1981) compared the closed-form and approximate solutions of R_t and indicated that the error would be high for a large f.

Approximations of the Arithmetic Average Roughness R_a

In the past, parabolic curves were used to approximate the elliptical or circular arcs to simplify the derivation of R_a. This is defined as the parabolic approximation. Assuming the surface profile consisting of parabolic curves with the same width (feed) and height (minor semi-axis) as the elliptical arcs, the approximate R_a can be expressed as

$$R_a \cong \frac{4}{9\sqrt{3}}\left(b - \frac{b}{a}\sqrt{a^2 - \frac{f^2}{4}}\right) \tag{25}$$

When $f^2 \ll ab$, a further simplified approximation can be obtained.

$$R_a \cong 0.032\frac{f^2 b}{a^2} \tag{26}$$

By substituting a and b by r, Eq. 26 turns to be $R_a \cong 0.032f^2/r$, which has been commonly used to estimate the R_a of a theoretical surface consisting of circular arcs, as reported in references (Cheung and Lee, 2001; Whitehouse, 1994).

Approximations of the Root-Mean-Square Roughness R_q

The derivation of R_q can also be simplified by approximating the elliptical arcs by parabolic curves with the same feed and height. The parabolic approximation of R_q is

$$R_q \cong \frac{2}{3\sqrt{5}}\left(b - \frac{b}{a}\sqrt{a^2 - \frac{f^2}{4}}\right) \tag{27}$$

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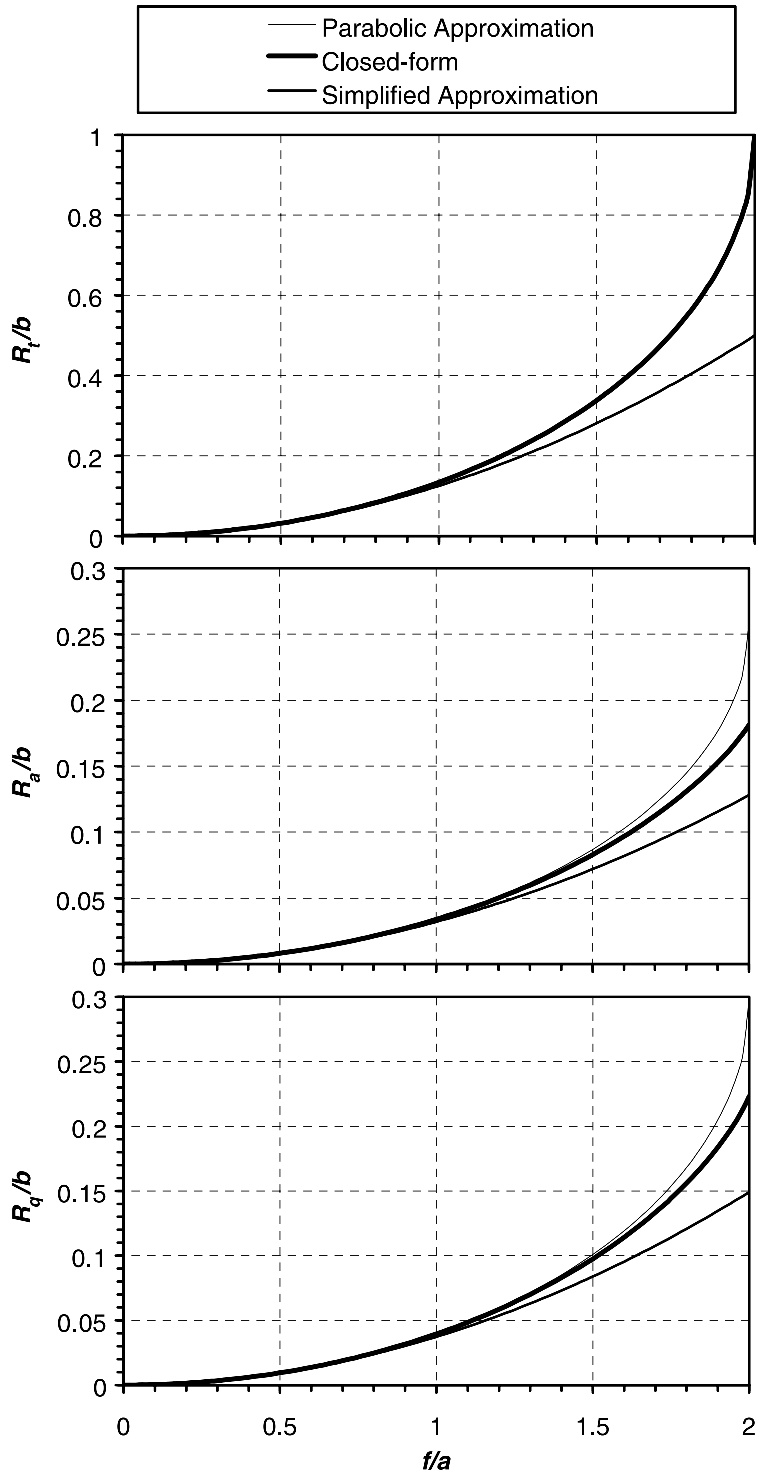

Figure 3. Comparison of closed-form and approximate solutions of surface finish parameters.

Table 1. Error analysis of approximate solutions.

	Error (%)				
	Parabolic approximation (overestimate)		Simplified approximation (underestimate)		
f/a	R_a	R_q	R_t	R_a	R_q
0.5	0.33%	0.23%	1.59%	1.50%	1.35%
1	1.53%	1.09%	6.70%	5.50%	5.67%
1.5	4.82%	3.47%	16.93%	13.13%	14.04%
2	41.53%	33.58%	50.00%	29.40%	33.21%

When $f^2 \ll ab$, the simplified approximation of R_q is

$$R_q \cong 0.037\frac{f^2 b}{a^2} \tag{28}$$

Comparison of the Closed-Form and Approximate Solutions

Results of the parabolic and simplified approximations of R_t, R_a, and R_q, comparing to the closed-form solution, are shown in Figure 3. The parabolic approximations overestimate and the simplified approximations underestimate the closed-form solution. Table 1 shows more detailed error analysis results for four different values of f/a. Under the worst case, when $f/a = 2$, the error is 41.5% and 33.6% for the parabolic approximation of R_a and R_q and 50.0%, 29.4%, and 33.2% for the simplified approximation of R_t, R_a and R_q, respectively. Very small error, less than 0.33% for the parabolic approximation and 1.6% for the simplified approximation, can be seen when $f/a = 0.5$. This indicates that the approximation solutions do provide good estimations of the surface roughness parameters when $f/a < 0.5$.

CONCLUSIONS

The closed-form solutions of surface finish parameters, R_t, R_a, and R_q, for the theoretical profile of a machined surface consisting of elliptical or circular arcs were derived. It offers accurate surface roughness evaluation for a wide range of process conditions and can be used in CAD/CAM systems for machining process modeling. The dimensionless form expressions of the three surface roughness parameters were also presented. Two approximation solutions, parabolic approximation and simplified approximation, were defined and developed to estimate the surface roughness parameters. Limitations of these approximate solutions were investigated. The comparison of the closed-form and approximate solutions showed that the parabolic approximation overestimated and the simplified approximation underestimated the surface roughness parameters. When f/a is smaller than 0.5, both approximation solutions have good estimation of R_t, R_a, and R_q.

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Although a detailed literature survey was conducted to investigate previous research in this subject, it is still possible that other researchers could have investigated this basic problem, perhaps in different format and other approaches. The goal of this study is to present a compete derivation of the closed-form solution from two different approaches and to provide a detailed comparison with two approximate solutions.

APPENDIX A. DERIVATION OF SEMI-AXES *a* AND *b* AND FEED *f* FOR A MEASUREMENT PROFILE ON THE SURFACE MACHINED BY FLAT-END MILLING

Figure A.1 shows a grooved surface machined by a flat-end mill with radius r and pitch p between parallel tool paths. The trace for surface finish measurement is at an angle θ relative to the velocity vector of the end mill, V. The definitions of the X- and Y-axes are the same as on the surface measurement profile presented in Figure 1. X-axis is tangent to the valley of the elliptical arcs and Y-axis is perpendicular to the nominal surface. On the X-Y plane, the theoretical profile consists of elliptical arcs with semi-axes a and b and feed f. Two angles, α and β, are used to define the orientation of the tool relative to the XYZ coordinate. A coordinate X′Y′Z′ with origin at O′ on the axis of the tool is illustrated in Figure A.1. X′, Y′, and Z′ axes are parallel to X, Y, and Z axes, respectively. Line O′D is the projection of the tool axis O′C on the X′-Z′ plane. α is the angle between X′-axis and O′D and β is the angle between O′C and O′D.

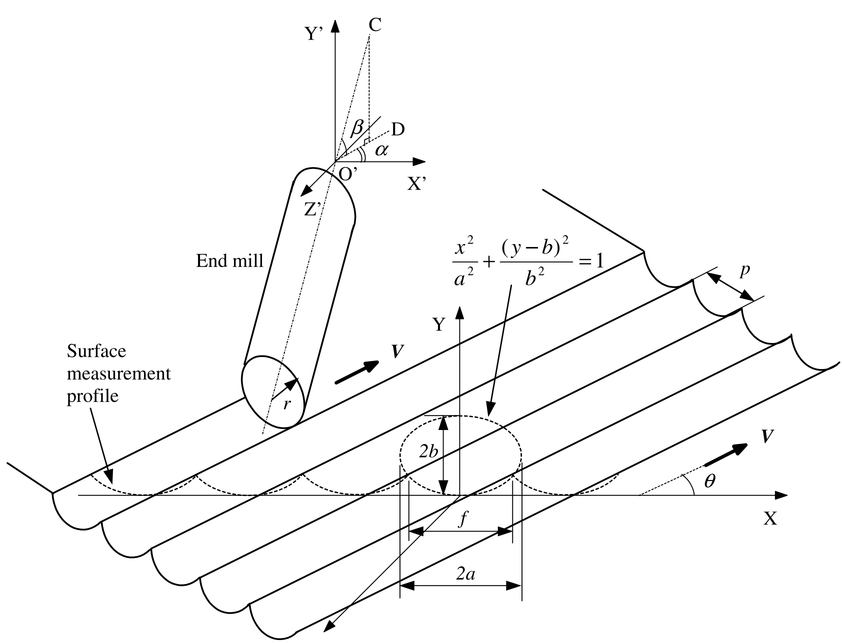

Figure A.1. Measurement profile on the surface machined by flat-end milling.

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The semi-axes a and b and feed f of the elliptical arc on the measuring profile are:

$$a = r\frac{\cos(\alpha - \theta)}{\sin\theta} \tag{A.1}$$

$$b = r\cos\beta \tag{A.2}$$

$$f = \frac{p}{\sin\theta} \tag{A.3}$$

APPENDIX B. DERIVATION OF THE CLOSED-FORM R_a AND R_q USING THE PARAMETRIC FORM

As shown in Figure 1, the elliptical arc, with a and b as the major and minor semi-axis and center at (0, b), on the surface profile can be expressed in the parametric form.

$$x = a\cos\theta \tag{B.1}$$

$$y = b\sin\theta + b \tag{B.2}$$

Define $S(\theta)$ as follows:

$$\begin{aligned} S(\theta) &= \int y dx = \int (b\sin\theta + b)(-a\sin\theta)d\theta \\ &= ab\left(\cos\theta + \frac{1}{4}\sin 2\theta - \frac{1}{2}\theta\right) \end{aligned} \tag{B.3}$$

Thus, $\bar{y}$ can be expressed as

$$\bar{y} = \frac{1}{f}\int_{-f/2}^{f/2} y dx = \frac{2}{f}\left(S(\theta_e) - \frac{\pi}{4}ab\right) \tag{B.4}$$

Two new parameters, θ_c and θ_e, are defined by $y(\theta_c) = \bar{y}$ and $x(\theta_e) = f/2$.

$$\theta_c = \arcsin\left(\frac{\bar{y}}{b} - 1\right) - \pi \tag{B.5}$$

$$\theta_e = -\arccos\left(\frac{f}{2a}\right) \tag{B.6}$$

The arithmetic average roughness R_a and root-mean-square roughness R_q of the theoretical profile then can be derived and represented by the following formulas.

$$\begin{aligned} R_a = \frac{1}{f}\int_{-f/2}^{f/2} |y - \bar{y}| dx &= \frac{1}{f^2}[8a\cos\theta_c \cdot S(\theta_e) - 2f \cdot S(\theta_c) \\ &\quad -2\pi a^2 b\cos\theta_c + \pi abf] \end{aligned} \tag{B.7}$$

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$$R_q = \sqrt{\frac{1}{f}\int_{-f/2}^{f/2}(y-\bar{y})^2 dx} = \sqrt{-\bar{y}^2 + 2b\bar{y} - \frac{f^2b^2}{12a^2}} \tag{B.8}$$

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REFERENCES

(1995). ASME B46.1. In: *Surface Texture—Surface Roughness, Waviness, and Lay*.

Cheung, C. F., Lee, W. B. (2001). Characterization of nanosurface generation in single-point diamond turning. *Int. J. Mach. Tools Manuf.* 41(6):851–875.

De-Chiffre, L., Lonardo, P., Trumpold, H., Lucca, D. A., Goch, G., Brown, C. A., Raja, J., Hansen, H. N. (2000). Quantitative characterization of surface texture. *Ann. CIRP* 49(2):635–652.

El-wardany, T., Elbestawi, M. A., Attia, M. H., Mohamed, E. (1992). Surface finish in turning of hardened steel. Engineered surfaces. *ASME Prod. Eng. Div.* 62:141–159.

Hasegawa, M., Seireg, A., Lindberg, R. A. (1976). Surface roughness model for turning. *Tribol. Int.* 9:285–289.

(1997). ISO 4287. In: *Geometrical Product Specifications (GPS)—Surface Texture: Profile Method—Terms, Definitions, and Surface Texture Parameters*.

Lambert, H. J. (1961–1962). Two years of finish-turning research at the Technological University, Delft. *Ann. CIRP* 10:246–255.

Lee, Y. S. (1998). Mathematical modeling of using different endmills and tool placement problems for 4- and 5-axis NC complex surface machining. *Int. J. Prod. Res.* 36(3):785–814.

Lee, Y. S., Chang, T. C. (1996). Machined surface error analysis for 5-axis machining. *Int. J. Prod. Res.* 34(1):111–135.

Malburg, M. C., Raja, J., Whitehouse, D. J. (1993). Characterization of surface texture generated by plateau honing process. *Ann. CIRP* 42(1):637–639.

Nassirpour, F., Wu, S. M. (1977). Statistical evaluation of surface finish and its relationship to cutting parameters in turning. *Int. J. Mach. Tool Des. Res.* 17:197–208.

Nowicki, B. (1981). Investigation of the surface roughness range. *Ann. CIRP* 30:493–497.

Olsen, K. V. (1968). Surface roughness on turned steel components and the relevant mathematical analysis. *Production* 61:593–606.

Qu, J., Shih, A. J., Scattergood, R. (2002). Development of the cylindrical wire electrical discharge machining process, Part II: surface integrity and roundness. *J. Manuf. Sci. Eng.* 124(3):708–714.

Sata, T. (1964). Surface finish in metal cutting. *Ann. CIRP* 7:190–197.

Sata, T., Li, M., Takata, S., Hiroaka, H., Li, C. Q., Xing, X. Z., Xiao, X. G. (1985). Analysis of surface roughness generation in turning operation and its applications. *Ann. CIRP* 34:473–476.

Shaw, M. C., Crowell, J. A. (1965). Finishing machining. *Ann. CIRP* 13:5–22.

Shiraishi, M., Sato, S. (1990). Dimensional and surface roughness controls in a turning operation. *J. Eng. Ind.* 112(1):78–83.

Thomas, T. R. (1981). Characterization of surface roughness. *Precis. Eng.* 3:97–104.

Vajpayee, S. (1973). Functional approach to numerical assessment of surface roughness. *Microtecnic* 27:360–361.

Vajpayee, S. (1981). Analytical study of surface roughness in turning. *Wear* 70:165–175.

Wallbank, J. (1979). Surfaces generated in single-point diamond turning. *Wear* 56:391–407.

Whitehouse, D. J. (1994). *Handbook of Surface Metrology*. Bristol, Philadephia: Institute of Physics.